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A new type of superstring in four dimensions

B.B. Deo

Physics Department, Utkal University, Bhubaneswar 751004, India

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Abstract

A bosonic string in twenty six dimensions is effectively reduced to four dimensions by eleven Majorana fermions which are vectors in the bosonic representation $SO(d-1, 1)$. By dividing the fermions in two groups, actions can be written down which are world sheet supersymmetric, 2d local and local 4d supersymmetric. The novel string is anomaly free, free of ghosts and the partition function is modular invariant.

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There are many ways to construct four-dimensional superstrings. Antoniadis, Bachas and Kounnas [1] have used trilinear coupling whereas Chang and Kumar [2] have taken Thirring coupling of fermions in addition to four bosonic coordinates. Kawai, Lewellen and Tye [3] have discussed the problem in a fairly general way. Consistent superstring as solutions of $D = 26$ bosonic string exists as has been shown by Casher, Englert, Nicolai and Taormina [4] and this reasoning has been extended by Englert, Houart and Taormina [5] to brane fusion of the bosonic string to form a fermionic string. We introduce fermions by noting that one boson is equivalent to two fermions. In a formal way, one can add 44 world sheet scalar fermions with internal symmetry $SO(44)$ to the four bosonic coordinates to obtain an equivalent Nambu Goto string in four dimensions. In this Letter we propose to effectively compactify the 26d string by 11 worldsheet vector fermions. There will be eleven metric ghosts which can be eliminated by eleven subsidiary conditions. The action in the world sheet (σ, τ) is

$$S = -\frac{1}{2\pi} \int d^2\sigma \left[\partial^\alpha X^\mu \partial_\alpha X_\mu - i \sum_{j=1}^{11} \bar{\phi}^{\mu,j} \rho^\alpha \partial_\alpha \phi_{\mu,j} \right]. \quad (1)$$

The internal symmetry of the ϕ^μ 's is $SO(11)$. They are real Majorana fermions which are vectors in the bosonic representation $SO(3, 1)$. If the action is to be supersymmetric, we must find a supersymmetric partner Ψ^μ to the bosonic coordinate X^μ . This can only be the linear sum of ϕ^μ 's like $\Psi^\mu = \sum_{j=1}^{11} e_\phi^j \phi_j^\mu$. With usual anticommutator following from action (1), the Ψ^μ will have the same anticommutator rule provided $\sum_{j=1}^{11} e_\phi^j e_{\phi j} = 1$. We searched

E-mail address: imaharan@iopb.res.in (B.B. Deo).

for such coefficients to find a linear combination which will make the action supersymmetric. After extensive and laborious probing we found only one possibility namely to group ϕ^μ 's into two species $\psi^{\mu,j}$; $j = 1, 2, \dots, 6$ and $\phi^{\mu,k}$; $k = 1, 2, \dots, 5$. For one group, the positive and negative frequency are $\psi^{\mu,j} = \psi^{(+)\mu,j} + \psi^{(-)\mu,j}$ where as for the other, allowing for the phase uncertainty of the creation operator for Majorana spinors, $\phi^{\mu,k} = \phi^{(+)\mu,k} - \phi^{(-)\mu,k}$. For the latter species, the equal time anticommutator has a negative sign. The action is taken as

$$S = -\frac{1}{2\pi} \int d^2\sigma [\partial_\alpha X^\mu \partial^\alpha X_\mu - i \bar{\psi}^{\mu,j} \rho^\alpha \partial_\alpha \psi_{\mu,j} + i \bar{\phi}^{\mu,k} \rho^\alpha \partial_\alpha \phi_{\mu,k}]. \quad (2)$$

The 2d Dirac matrices are

$$\rho^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad (3)$$

and

$$\rho^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \quad (4)$$

and $\bar{\psi} = \psi^+ \rho^0$, $\bar{\phi} = \phi^+ \rho^0$. It is worthwhile noting that the light cone fermionic action

$$S_{\text{l.c.}} = \frac{i}{2\pi} \int d^2\sigma \sum_{\mu=0,3} (\bar{\psi}^{\mu,j} \rho^\alpha \partial_\alpha \psi_{\mu,j} - \bar{\phi}^{\mu,k} \rho^\alpha \partial_\alpha \phi_{\mu,k})$$

has central charge 11 and $S - S_{\text{l.c.}}$ has central charge 15 as the usual $N = 1$, supersymmetric action. In this model, $S_{\text{l.c.}}$ is the equivalent of the Faddeev–Popov superconformal ghost action of the superstring. It is necessary to introduce two unit vectors \mathbf{e}_ψ and \mathbf{e}_ϕ with eleven components, e.g., for $j = 3$, and $k = 3$,

$$\mathbf{e}_\psi^3 = (0, 0, 1, 0, 0, 0; 0, 0, 0, 0, 0), \quad \mathbf{e}_\phi^3 = (0, 0, 0, 0, 0, 0; 0, 0, 1, 0, 0) \quad (5)$$

with properties $e_\psi^j e_{\psi n} = \delta_n^j$ and $e_\phi^k e_{\phi m} = \delta_m^k$. We shall frequently use $e_\psi^j e_{\psi j} = 6$ and $e_\phi^k e_{\phi k} = 5$. Everywhere the upper index refers to a column and the lower to a row. The suffixes ψ and ϕ are for bookkeeping only. The action S in (2) is found to be invariant under the following infinitesimal supersymmetric transformation.

$$\delta X^\mu = \bar{\epsilon} (e_\psi^j \psi_j^\mu - e_\phi^k \phi_k^\mu), \quad \delta \psi^{\mu,j} = -i e_\psi^j \rho^\alpha \partial_\alpha X^\mu \epsilon \quad (6)$$

and

$$\delta \phi^{\mu,k} = i e_\phi^k \rho^\alpha \partial_\alpha X^\mu \epsilon \quad (7)$$

with ϵ is the infinitesimal constant anticommutating spinor. The commutator of two transformations is the translation

$$[\delta_1, \delta_2] X^\mu = a^\alpha \partial_\alpha X^\mu, \quad [\delta_1, \delta_2] \psi^{\mu,j} = a^\alpha \partial_\alpha \psi^{\mu,j} \quad (8)$$

and

$$[\delta_1, \delta_2] \phi^{\mu,k} = a^\alpha \partial_\alpha \phi^{\mu,k}, \quad (9)$$

where $a^\alpha = 2i \bar{\epsilon}_1 \rho^\alpha \epsilon_2$. Eq. (9) is true only and only if

$$\psi_j^\mu = e_{\psi j} \Psi^\mu \quad (10)$$

and

$$\phi_k^\mu = e_{\phi k} \Psi^\mu. \quad (11)$$

The Majorana fermion

$$\Psi^\mu = e_\psi^j \psi_j^\mu - e_\phi^k \phi_k^\mu \quad (12)$$

is such that

$$\delta X^\mu = \bar{\epsilon} \Psi^\mu, \quad \delta \Psi^\mu = -i \epsilon \rho^\alpha \partial_\alpha X^\mu \quad (13)$$

and

$$[\delta_1, \delta_2] \Psi^\mu = a^\alpha \partial_\alpha X^\mu \quad (14)$$

and is the right candidate to be the supersymmetric partner of the bosonic coordinates X^μ and also is the desired sum of ψ^μ . The action in (1) is only world sheet supersymmetric, but not local 2d supersymmetric.

Besides the above pair, with the introduction of another pair, the zweibein $e_\alpha(\sigma, \tau)$ and the two-dimensional spinor-cum-world sheet vector χ_α , $\delta \chi_\alpha = \nabla_\alpha \epsilon$ the local 2d supersymmetric invariant action is [6]

$$S = -\frac{1}{2\pi} \int d^2\sigma e \left[h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu - i \bar{\Psi}^\mu \rho^\alpha \partial_\alpha \Psi_\mu + 2 \bar{\chi}_\alpha \rho^\beta \rho^\alpha \Psi^\mu \partial_\beta X_\mu + \frac{1}{2} \bar{\Psi}^\mu \Psi_\mu \bar{\chi}_\beta \rho^\beta \rho^\alpha \chi_\alpha \right]. \quad (15)$$

This is necessary to derive the vanishing of the energy–momentum tensor and Noether current from a gauge principle [6,11]. Varying the field and the zweibein, we derive the vanishing of the supercurrent J_α and the energy–momentum tensor $T_{\alpha\beta}$ on the world sheet,

$$J_\alpha = \frac{\pi}{2e} \frac{\delta S}{\delta \bar{\chi}^\alpha} = \rho^\beta \rho_\alpha \bar{\Psi}^\mu \partial_\beta X_\mu - \frac{1}{4} \bar{\Psi}^\mu \Psi_\mu \rho_\alpha \rho^\beta \chi_\beta = 0. \quad (16)$$

The gravitino field χ_α decouples and in the gauge $\chi_\alpha = 0$, we have

$$J_\alpha = \rho^\beta \rho_\alpha \bar{\Psi}^\mu \partial_\beta X_\mu = 0. \quad (17)$$

Variation of action with respect to ‘zweibein’ e_α^a gives $T_\beta^a = -\frac{\pi}{2e} \frac{\delta S}{\delta e_\alpha^a} = 0$ and implies $T_{\alpha\beta} = e_{\alpha a} T_\beta^a$. The variation with respect to the factor ‘ e ’ gives the traceless part. Since $h^{\alpha\beta} = e_\alpha^a e_\beta^b \eta^{ab}$, the bosonic term is obtained by finding the variation $\frac{\delta S}{\delta h^{\alpha\beta}}$. Due to spin connection ∇_α of the fermionic part is simply ∂_α and $\rho^\alpha \partial_\alpha$ should be written as $\rho^a e_a^\beta \partial_\beta$ so that the variation gives $S^F \sim \frac{i\epsilon}{2} \bar{\Psi}^\mu \rho^a e_a^\alpha \partial_\alpha \Psi_\mu$ leading to $\frac{\delta S^F}{\delta e_\alpha^a} \sim \frac{i\epsilon}{2} \bar{\Psi}^\mu \rho^a \partial_\alpha \Psi_\mu$. The result is

$$T_{\alpha\beta} = -\frac{\pi}{2e} e_{\alpha a} T_\beta^a = \partial_\alpha X^\mu \partial_\beta X_\mu - \frac{i}{2} \bar{\Psi}^\mu \rho_{(\alpha} \partial_{\beta)} \Psi_\mu - (\text{trace}) = 0. \quad (18)$$

In a basis $\tilde{\psi} = (\psi^+, \psi^-)$ and $\tilde{\phi} = (\phi^+, \phi^-)$ the vanishing of the light cone components are

$$J_\pm = \partial_\pm X_\mu \Psi_\pm^\mu \quad (19)$$

and

$$T_{\pm\pm} = \partial_\pm X^\mu \partial_\pm X_\mu + \frac{i}{2} \psi_\pm^{\mu j} \partial_\pm \psi_{\pm\mu, j} - \frac{i}{2} \phi_\pm^{\mu k} \partial_\pm \phi_{\pm\mu, k}, \quad (20)$$

where $\partial_\pm = \frac{1}{2}(\partial_\tau \pm \partial_\sigma)$. The component constraints are the following equations (19)

$$\partial_\pm X_\mu \psi_\pm^{\mu, j} = \partial_\pm X_\mu e_\psi^j \Psi_\pm^\mu = 0, \quad j = 1, 2, \dots, 6, \quad (21)$$

$$\partial_\pm X_\mu \phi_\pm^{\mu k} = \partial_\pm X_\mu e_\phi^k \Psi_\pm^\mu = 0, \quad k = 1, 2, \dots, 5. \quad (22)$$

These eleven constraints eliminate the eleven Lorentz metric ghosts from the physical spectrum. However, they follow from one current constraint of Eq. (19). Besides the conformal ghosts (b, c), we shall need eleven pairs of

the superconformal (β^j, γ^j) and (β^k, γ^k) ghosts to write down BRST charge. These eleven pairs behave like one pair (β, γ) , like the current generator. One can easily construct ‘null’ physical states as done in Ref. [6].

For the two groups of fermions the general constraints are the two equations

$$\partial_{\pm} X_{\mu} e_{\phi}^k \phi_{\pm k}^{\mu} = \partial_{\pm} X_{\mu} e_{\psi}^j \psi_{\pm j}^{\mu} = 0. \quad (23)$$

However, action given in Eq. (15) is not invariant under four-dimensional local supersymmetry. To obtain the simplest Green–Schwarz action for $N = 1$ local supersymmetry, we note that there are four component Dirac spinor representations of $SO(3, 1)$ which we denote by $\theta_{j\delta}$ and $\theta_{k\delta}$, respectively, with $\delta = 1, 2, 3, 4$. A genuine spacetime fermion can be constructed rather than spacetime vector as

$$\theta_{\delta} = \sum_{j=1}^6 e_{\psi}^j \theta_{j\delta} - \sum_{k=1}^5 e_{\phi}^k \theta_{k\delta}. \quad (24)$$

The G.S. action is [6] for $N = 1$

$$S = \frac{1}{2\pi} \int d^2\sigma (\sqrt{g} g^{\alpha\beta} \Pi_{\alpha} \Pi_{\beta} + 2i\epsilon^{\alpha\beta} \partial_{\alpha} X^{\mu} \bar{\theta} \Gamma_{\mu} \partial_{\beta} \theta), \quad (25)$$

where Γ_{μ} are the Dirac gamma matrices and

$$\Pi_{\alpha}^{\mu} = \partial_{\alpha} X^{\mu} - i\bar{\theta} \Gamma^{\mu} \partial_{\alpha} \theta.$$

The action is spacetime supersymmetric in four dimensions [6]. We have traded the vector four indices μ for the four spinor components δ to form the genuine spacetime Dirac spinors. The major defect is that the naive covariant quantisation does not work. To proceed with covariant formulation we have to implement NS–R [7] scheme with G.S.O. [8] projection which is simple, elegant and equivalent. The non-vanishing equal time commutator and anti-commutators follow from the action in Eq. (1)

$$[\partial_{\pm} X^{\mu}(\sigma, \tau), \partial_{\pm} X^{\nu}(\sigma', \tau)] = \pm \frac{\pi}{2} \eta^{\mu\nu} \delta'(\sigma - \sigma') \quad (26)$$

and

$$\{\psi_A^{\mu}(\sigma), \psi_B^{\nu}(\sigma')\} = \pi \eta^{\mu\nu} \delta'(\sigma - \sigma') \delta_{AB}. \quad (27)$$

Even though

$$\{\phi_A^{\mu}(\sigma), \phi_B^{\nu}(\sigma')\} = -\pi \eta^{\mu\nu} \delta'(\sigma - \sigma') \delta_{AB}, \quad (28)$$

A, B refer to the component indices 1, 2 or the helicities ‘+’, ‘−’. There are no ghost quanta other than $\mu = \nu = 0$ due to negative phase of creation operator. We immediately check that

$$[T_{++}(\sigma), T_{++}(\sigma')] = i\pi \delta'(\sigma - \sigma') [T_{++}(\sigma) + T_{++}(\sigma')], \quad (29)$$

$$[T_{++}(\sigma), J_+(\sigma')] = i\pi \delta'(\sigma - \sigma') \left[J_+(\sigma) + \frac{1}{2} J_+(\sigma') \right] \quad (30)$$

and

$$[J_+(\sigma), J_+(\sigma')] = \pi \delta'(\sigma - \sigma') T_{++}(\sigma). \quad (31)$$

Similarly for ‘−’, ‘−’ components. The superalgebra closes.

Let α_m^{μ} denote the quanta of coordinates satisfying

$$[\alpha_m^{\mu}, \alpha_n^{\nu}] = m \delta_{m, -n} \eta^{\mu\nu}. \quad (32)$$

With NS conditions, let b_r^μ and $b_r'^\mu$ be the quanta of ψ and ϕ fermions with half integral r where as with Ramond boundary conditions they are d_m^μ and $d_m'^\mu$ with m integers. The non-vanishing anticommutator relations are

$$\{b_r^{\mu,j}, b_s^{\nu,j'}\} = \eta^{\mu\nu} \delta^{j,j'} \delta_{r,-s}, \quad \{b_r'^{\mu,k}, b_s'^{\nu,k'}\} = -\eta^{\mu\nu} \delta^{k,k'} \delta_{r,-s} \quad (\text{NS}), \quad (33)$$

$$\{d_m^{\mu,j}, d_n^{\nu,j'}\} = \eta^{\mu\nu} \delta^{j,j'} \delta_{m,-n}, \quad \{d_m'^{\mu,k}, d_n'^{\nu,k'}\} = -\eta^{\mu\nu} \delta^{k,k'} \delta_{m,-n} \quad (\text{R}), \quad (34)$$

with

$$b_{-r,j}'^\mu = -b_{r,j}^{\prime\ddagger\mu} \quad \text{and} \quad d_{-m,j}'^\mu = -d_{m,j}^{\prime\ddagger\mu}, \quad (35)$$

It will be useful to note $b_r^{\mu,j} = e_\psi^j B_r^\mu$, $b_r'^{\mu,k} = e_\phi^k B_r^\mu$, $B_r^\mu = e_\psi^j b_{r,j}^\mu - e_\phi^k b_{r,k}'^\mu$. Similarly for the d , d' ; $D_m^\mu = e_\psi^j d_{m,j}^\mu - e_\phi^k d_{m,k}'^\mu$.

Virasoro generators [9]

$$L_m = \frac{1}{\pi} \int_{-\pi}^{\pi} d\sigma e^{im\sigma} T_{++}, \quad (36)$$

$$G_r = \frac{\sqrt{2}}{\pi} \int_{-\pi}^{\pi} d\sigma e^{ir\sigma} J_+ \quad (\text{NS}), \quad (37)$$

$$F_n = \frac{\sqrt{2}}{\pi} \int_{-\pi}^{\pi} d\sigma e^{in\sigma} J_+ \quad (\text{R}), \quad (38)$$

$$[L_m, L_n] = (m-n)L_{m+n} + A(m)\delta_{m,-n}, \quad (39)$$

$$[L_m, G_r] = \left(\frac{1}{2}m - r\right)G_{m+r} \quad (\text{NS}), \quad (40)$$

$$\{G_r, G_s\} = 2L_{s+r} + B(r)\delta_{r,-s}, \quad (41)$$

$$[L_m, F_n] = \left(\frac{1}{2}m - n\right)F_{m+n} \quad (\text{R}), \quad (42)$$

$$\{F_m, F_n\} = 2L_{m+n} + B_F(m)\delta_{m,-n}. \quad (43)$$

Here $A(m)$, $B(r)$ and $B_F(m)$ are normal ordering constants, since the generators are the sum of the products of normal ordered quantum operators ‘:.’. A single dot implies sum over all qualifying indices.

$$\text{NS: } L_m = \frac{1}{2} \sum_{-\infty}^{\infty} : \alpha_{-n} \alpha_{m+n} : + \frac{1}{2} \sum_{-\infty}^{\infty} \left(r + \frac{1}{2}m\right) : (b_{-r} b_{m+r} - b_{-r}' b_{m+r}') :, \quad (44)$$

$$G_r = \sum_{-\infty}^{\infty} \alpha_{-n} B_{n+r}, \quad (45)$$

$$\text{R: } L_m = \frac{1}{2} \sum_{-\infty}^{\infty} : \alpha_{-n} \alpha_{m+n} : + \frac{1}{2} \sum_{-\infty}^{\infty} \left(n + \frac{1}{2}m\right) : (d_{-n} d_{m+n} - d_{-n}' d_{m+n}') :, \quad (46)$$

$$F_m = \sum_{-\infty}^{\infty} \alpha_{-n} D_{n+m}. \quad (47)$$

Both in NS and R sectors, there are the light cone ghosts corresponding to the fermionic components $\mu = 0$ and 3 which are present in L_m . By standard technique, we calculate that

$$A(m) = \frac{26}{12}(m^3 - m) = \frac{C}{12}(m^3 - m). \quad (48)$$

$C = 26$ is the central charge. Using Jacobi identity

$$B(r) = \frac{1}{2r}A(r) = \frac{26}{3}\left(r^2 - \frac{1}{4}\right) = \frac{C}{3}\left(r^2 - \frac{1}{4}\right). \quad (49)$$

The central charge C can also be deduced from the v.e.v. of the product of two energy–momentum tensors, e.g.,

$$\langle T_+(z)T_+(\omega) \rangle \sim \frac{C}{2}(z - \omega)^{-4} + \dots \quad (50)$$

with

$$C = \eta_\mu^\mu + \frac{1}{2}\eta_\mu^\mu \delta_j^j + \frac{1}{2}\eta_\mu^\mu \delta_k^k = 26. \quad (51)$$

The normal order anomaly in Ramond sector must note that F_0 has no ambiguity. So $F_0^2 = L_0$. Using the algebraic relation for F 's and the Jacobi identity

$$\{F_r, F_{-r}\} = \frac{2}{r}\{[L_r, F_0], F_{-r}\} = L_0 + \frac{4}{r}A(r) \quad (52)$$

leads to

$$B_R(m) = \frac{4}{m}A(m) = \frac{C}{3}(m^2 - 1); \quad m \neq 0. \quad (53)$$

The anomalies are have to be cancelled out and one needs the Faddeev–Popov ghosts $b_{\pm\pm}$, c^\pm fields given by the action

$$S_{\text{FP}} = \frac{1}{\pi} \int d^2\sigma (c^+ \partial_- b_{++} + c^- \partial_+ b_{--}). \quad (54)$$

The c^\pm , $b_{\pm\pm}$ field quanta b_n , c_n satisfy the anticommutator relation $\{c_m, b_n\} = \delta_{m,-n}$ and $\{c_m, c_n\} = \{b_n, b_m\} = 0$. The Virasoro generators for the ghost become

$$L_m^{\text{gh}} = \sum_{n=-\infty}^{\infty} (m-n)b_{m+n}c_{-n} - a\delta_{m,0}, \quad (55)$$

where a is the normal ordering constant. If G_r^{gh} and F_r^{gh} are the ghost current generators of conformal dimension $\frac{3}{2}$, the super-Virasoro algebras for the ghosts for the two sectors follow. The anomaly term is deduced to be

$$A^{\text{gh}}(m) = \frac{1}{6}(m - 13m^3) + 2am = -\frac{26}{12}(m^3 - m) + 2(a - 1)m. \quad (56)$$

To be anomaly free, $A^{\text{gh}}(m) + A(m) = 0$. This gives the value of $a = 1$ for the normal ordering constant and -26 for the central charge of the ghost. Due to Jacobi identity, the anomaly terms for the ghosts for both the sectors cancels as well. Superconformal ghosts which contributes central charge 11 in a normal superstring are not necessary. To explain this we observe that the light cone gauge is ghost free. Dropping the helicity suffixes, the lightcone vectors, $\psi_j^\pm = \frac{1}{\sqrt{2}}(\psi_j^0 \pm \psi_j^3)$ and $\phi_j^\pm = \frac{1}{\sqrt{2}}(\phi_j^0 \pm \phi_j^3)$ have the anticommutators with negative sign and are the ghost modes. The total ghost energy–momentum tensor comes from these (0, 3) coordinates.

$$T^{\text{gh}}(z) = \frac{i}{2}(\psi^{0j} \partial_z \psi_{0j} + \psi^{3j} \partial_z \psi_{3j}) - \frac{i}{2}(\phi^{0k} \partial_z \phi_{0k} + \psi^{3k} \partial_z \psi_{3k}). \quad (57)$$

But the vacuum correlation function is

$$\langle T^{\text{gh}}(z) T^{\text{gh}}(\omega) \rangle = \frac{11}{2} (z - \omega)^{-4} + \dots \quad (58)$$

Thus the contribution of these ghosts to the central charge 26 is 11 like the superconformal ghosts. Without these ghosts the central charge is 15 as is the case for normal ten-dimensional superstring. With no anomalies and ghosts, the physical states can be constructed as

$$\text{Bosonic NS: } (L_0 - 1)|\Psi\rangle = 0, \quad L_m|\Psi\rangle = 0, \quad G_r|\Psi\rangle = 0, \quad m, r > 0, \quad (59)$$

$$\text{Fermionic R: } (L_0 - 1)|\Phi\rangle_\alpha = 0, \quad L_m|\Phi\rangle_\alpha = 0, \quad F_m|\Phi\rangle_\alpha = 0, \quad m > 0. \quad (60)$$

Since $L_0 = F_0^2$, we also have $(F_0 - 1)|\phi_+\rangle_\alpha = 0$ and $(F_0 + 1)|\phi_-\rangle_\alpha = 0$.

We proceed to prove that these physical state conditions eliminate all the ghosts. We begin with Gupta Bleuler postulate $\alpha_{-1}^0|\Phi\rangle = 0$. From the physical state condition for L_m , $[L_{m+1}, \alpha_{-1}^0]|\Phi\rangle = \alpha_m^0|\Phi\rangle = 0$. The longitudinal modes of the coordinates drop out. For the bosonic NS sector, the physical state condition for B_r is used. $[B_{r+1}, \alpha_{-1}^0]|\Phi\rangle = B_r^0|\Phi\rangle = 0$. By definition, $b_r^{j,0}|\Phi\rangle = e_\psi^j B_r^0|\Phi\rangle = 0$. Similarly, $b_r'^{k,0}|\Phi\rangle = e_\phi^k B_r^0|\Phi\rangle = 0$. In the fermionic R sector, the physical state condition for F_m , leads to $[F_{m+1}, \alpha_{-1}^0]|\Phi\rangle = D_m^0|\Phi\rangle = 0$. As before, by definition, $d_m^{k,0}|\Phi\rangle = d_m'^{k,0}|\Phi\rangle = 0$. Thus the longitudinal modes of all the quanta are absent in the Fock space. Using the usual pair of (β, γ) commuting ghosts, the nilpotent BRST charge operator

$$Q_{\text{BRST}} = \sum L_{-m} c_m - \frac{1}{2} \sum (m-n) :c_{-m} c_{-n} b_{m+n}: - c_0 + Q' \quad (61)$$

where L_m is given by Eqs. (44), (46) and

$$Q' = \sum G_{-r} \gamma_r - \sum \gamma_{-r} \gamma_{-s} b_{r+s}, \quad \text{for NS}, \quad (62)$$

$$Q' = \sum F_{-n} \gamma_n - \sum \gamma_{-n} \gamma_{-m} b_{n+m}, \quad \text{for R}. \quad (63)$$

Q' takes care of the constraints due to the vanishing of the current generator operators acting on the physical states (59) and (60). In constructing the nilpotent charge we strictly followed the procedure for construction from operators satisfying Lie and graded Lie algebra [6]. It is to be noted that $Q_{\text{BRST}} - Q'$ and Q_{BRST} are separately nilpotent.

Actually there are eleven subsidiary physical state conditions like (59) and (60) having the property following the Fourier transformation and definition

$$G_r^j|\Phi\rangle = e_\psi^j G_r|\Phi\rangle = 0, \quad G_r^k|\Phi\rangle = e_\phi^k G_r|\Phi\rangle = 0, \quad (\text{NS}) \quad (64)$$

$$F_m^j|\Phi\rangle = e_\psi^j F_m|\Phi\rangle = 0, \quad F_m^k|\Phi\rangle = e_\phi^k F_m|\Phi\rangle = 0, \quad (\text{R}), \quad (65)$$

with $G_r = e_\psi^j G_{r,j} - e_\phi^k G_{r,k}$ and $F_m = e_\psi^j F_{m,j} - e_\phi^k F_{m,k}$. We can follow up by writing (γ_r^j, β_r^j) and (γ_r^k, β_r^k) with similar definitions. But, in the NS sector, for instance, $G_{-r}^j \gamma_{r,j} - G_{-r}^k \gamma_{r,k} \equiv G_{-r} \gamma_r$ where $\gamma_r = e_\psi^j \gamma_{r,j} - e_\phi^k \gamma_{r,k}$. So that the expressions for Q and Q' contain the conformal ghosts (b, c) and the pair of (β, γ) super conformal ghosts unitedly allowing for a BRST charge $Q_{\text{BRST}}^2 = 0$. The theory is ghost free and unitary.

L_0 , being the Hamiltonian, we obtain the mass spectrum of the model

$$\text{NS: } \alpha' M^2 = -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, \frac{3}{2}, \dots \quad (66)$$

and

$$\text{R: } \alpha' M^2 = -1, 0, 1, 2, 3, \dots \quad (67)$$

The self energy of the scalar tachyonic vacuum is cancelled by the contribution of the fermionic loop of the Ramond sector because the self energy of the fermion loop is $-\langle 0|(F_0 - 1)^{-1}(F_0 + 1)^{-1}|0\rangle = -\langle 0|(L_0^{\text{NS}} - 1)^{-1}|0\rangle$. The half integral values are eliminated by the clever G.S.O. projection of the states.

$$G = \frac{1}{2}(1 + (-1)^{F+F'}) \quad (68)$$

where the F and F' are the fermionic numbers of b and b' quanta. Thus both the fermions and bosons lie on linear Regge trajectories $\alpha' M^2 = -1, 0, 1, 2, 3, \dots$. The G.S.O. projection is necessary and essential to prove the modular invariance of the partition function of this string.

In covariant formulation, the effective number of the physical modes is the total number of modes minus the number of invariant constraints. Here there are forty four fermions and four constraints given by Eq. (23). So there are forty spacetime fermions in the assembly. Let us cast them in SO(6) and SO(5) invariant way. Let $b_{r,\alpha}^k$ be the annihilation quanta. α runs from 1 to 8 and k from 1 to 5. In the NS sector the Hamiltonian $H^{\text{NS}} = \sum_{k=1}^5 H_k^{\text{NS}}$ where $H_k^{\text{NS}} = \sum_r b_{r,\alpha}^{k\dagger} b_{r,\alpha}^k - \frac{8}{48}$. But $[H_k^{\text{NS}}, H_{k'}^{\text{NS}}] = 0$. The partition function of the forty member assembly will be the partition function of eight fermions raised to the power of five. The path integral function for the eight fermions is given by Seiberg and Witten [10]. In their notation

$$A_k((-), \tau) = \left(\frac{\Theta_3(\tau)}{\eta(\tau)} \right)^4, \quad (69)$$

$$A_k((+), \tau) = A_k\left((-), \frac{\tau}{(1+\tau)}\right) = -\left(\frac{\Theta_2(\tau)}{\eta(\tau)} \right)^4, \quad (70)$$

$$A_k((-+), \tau) = A_k\left((+-), \frac{-1}{\tau}\right) = -\left(\frac{\Theta_4(\tau)}{\eta(\tau)} \right)^4, \quad (71)$$

$$A_k(++), \tau) = 0. \quad (72)$$

The sum of all spin structure is

$$A_k(\tau) = \frac{\Theta_3^4(\tau) - \Theta_2^4(\tau) - \Theta_4^4(\tau)}{\eta^4(\tau)}. \quad (73)$$

The total partition function $Z = |A_k(\tau)|^{10}$ is not only modular invariant but vanishes due to the famous Jacobi relation which is the necessary condition for space time supersymmetry.

Thus we have been able to construct a novel modular invariant superstring which is free of anomalies and ghosts. Novel features of the model are that this is the only superstring known and constructed whose central charge is 26 and there is no need for superconformal ghosts in covariant formulation except in the construction of BRST charge. These ghosts are usually contributing 11 to the central charge and are replaced by 11 Lorentz metric ghost of vector fermions. This linkage needs further study. There are useful and harmless tachyons in NS and R sector, but the self energies of the tachyonic vacua cancel. This cancellation mechanism, perhaps, makes the vacua unique. Since bosonic string theory is simpler and well understood in general, a derivation of the four-dimensional spacetime supersymmetric action will positively help in studying the dynamics of the superstring theory in greater depth and insight.

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